Modeling of Frequency-Dependent Magnetized Plasma in Hybrid Symmetrical Condensed TLM Method

M. I. Yaich, M. Khalladi, I. Zekik, and J. A. Morente

***Abstract*—In this paper, magnetized plasma media are modeled using the time-domain TLM method with hybrid symmetrical condensed node (HSCN). The proposed technique consists in adding voltage sources characterizing dispersive media in the HSCN. Numerical results are presented for reflection and transmission coefficients for a magnetized plasma wall, proving the efficiency of the proposed model.**

***Index Terms*—Magnetized plasma materials, time-domain electromagnetics, TLM method.**

I. INTRODUCTION

THE TLM method is a robust and efficient numerical tool to model electromagnetic problems in the time domain. It has been successfully used for the analysis of numerous electromagnetic problems [1]–[8]. This method is able to simulate EM waves propagation and scattering by objects of a complex geometry and constant physical properties [1]–[3]. However, when the material is dispersive, this method requires an additional treatment to include frequency-dependent material properties in the time domain [4]–[8].

In order to model dispersive media in the time domain using the TLM method, three recent approaches have been proposed. The first one, proposed by Menezes and Hoefer, decouples the pulse scattering at the symmetrical condensed node (SCN) using equations describing the medium behavior in terms of equivalent node sources [4]. The second one is a systematic tool for the analysis of general properties media in the time domain using the Z-transform methods [5], [6]. This technique, has been extensively used by Sullivan to model EM waves propagation in dispersive media using the FDTD [9]–[11]. The third approach makes use of the equivalence between Maxwell equations and transmission line equations in order to model linear frequency-dependent isotropic materials by means of hybrid symmetrical condensed node (HSCN) TLM method [7], [8].

In this paper, the latter approach is extended to model gyrotropic plasma media. The expressions of the voltage sources describing the behavior of this dispersive media have been deduced from the analogy between the EM field magnitudes and the circuit magnitudes. The TLM approach using the HSCN and voltage sources is validated by the study of EM waves reflection and transmission through a magnetized plasma layer [6], [12], [13].

II. FORMULATION

In the case of magnetized plasma, the relationship between the electric field and the electric displacement is given by

$$D_i(\omega) = \hat{\varepsilon}_{ij}(\omega) E_j(\omega) \tag{1}$$

where $\hat{\varepsilon}_{ij}(\omega)$ is the tensor permittivity.

In the time domain, the above equation becomes a convolution involving susceptibility functions. These convolutions can be evaluated recursively if the complex susceptibility functions are introduced. However, only the real parts of these functions are used to update the electric field in the numerical resolution algorithms [12], [13].

Assuming that the EM wave propagates along the z axis and that the electric field remains constant during the time interval Δt, the recursive convolution method, based on the discretization of the convolution integral, yields the electric field components at the instant $(k+1)\Delta t$: [see (2) at the bottom of the next page] where

$$C_{xy} = \frac{\chi_{xy}^0}{1+\chi_{xx}^0} \tag{3}$$

$$C_{yx} = \frac{\chi_{yx}^0}{1+\chi_{yy}^0} \tag{4}$$

$$\alpha_x = \frac{\chi_{xx}^{0^2} + \chi_{xx}^0 + \chi_{xy}^{0^2}}{(1+\chi_{xx}^0)^2 + \chi_{xy}^{0^2}} \tag{5}$$

$$\alpha_y = \frac{\chi_{yy}^{0^2} + \chi_{yy}^0 + \chi_{yx}^{0^2}}{(1+\chi_{yy}^0)^2 + \chi_{yx}^{0^2}} \tag{6}$$

$$\alpha_z = \frac{\chi_{zz}^0}{1+\chi_{zz}^0} \tag{7}$$

$$\beta_x = \frac{1+\chi_{xx}^0}{(1+\chi_{xx}^0)^2 + \chi_{xy}^{0^2}} \tag{8}$$

$$\beta_y = \frac{1+\chi_{yy}^0}{(1+\chi_{yy}^0)^2 + \chi_{yx}^{0^2}} \tag{9}$$

$$\beta_z = \frac{1}{1+\chi_{zz}^0} \tag{10}$$

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${\chi_{uv}}^0$, with $(u,v) \in \{x,y\}$ and ${\chi_{zz}}^0$, are the real susceptibility functions and, finally, the discrete convolutions at time $k\Delta t$ of (2) are defined by

$$\psi_{uv}^k = \mathrm{Re}\left[\sum_{m=0}^{k-1} \Delta\hat{\chi}_{uv}^m \cdot {}_{k-m}E_v\right], \quad (u,v) \in \{x,y\} \tag{11}$$

and

$$\psi_{zz}^k = \mathrm{Re}\left[\sum_{m=0}^{k-1} \Delta\hat{\chi}_{zz}^m \cdot {}_{k-m}E_z\right] \tag{12}$$

where Re is the real operation, the $\hat{}$ designates complex quantities, $\Delta\hat{\chi}_{uv}^k = \hat{\chi}_{uv}^k - \hat{\chi}_{uv}^{k+1}$, $(u,v) \in \{x,y\}$, and $\Delta\hat{\chi}_{zz}^k = \hat{\chi}_{zz}^k - \hat{\chi}_{zz}^{k+1}$ [12], [13].

In order to obtain a set of equations compatible with the TLM method using the HSCN and voltage sources, we begin by enforcing charge conservation condition in the HSCN [1]. Then, we use the equations relating incident and reflected pulses for the EM field components so as to obtain the equivalent voltages in the node at the time $(k+1)\Delta t$ [14]. Finally, we couple the equivalent voltages components located in the xoy plane to obtain the following expressions: [see (13) at the bottom of the page] where $(V_{svx}, V_{svy}, V_{svz})$ are the voltage sources at ports (16, 17, 18) of the HSCN, (Y_{ox}, Y_{oy}, Y_{oz}) are the normalized admittances of the HSCN capacitive stubs (13,14,15), and (C_{xy}, C_{yx}) are the coupling coefficients.

The analogy between the sets of (2) and (13) is used to deduce the following expressions of the voltage sources:

$$\begin{pmatrix} {}_{k+1}V_{svx} + {}_kV_{svx} \\ {}_{k+1}V_{svy} + {}_kV_{svy} \\ {}_{k+1}V_{svz} + {}_kV_{svz} \end{pmatrix} = \begin{pmatrix} 4+Y_{ox} \\ 4+Y_{oy} \\ 4+Y_{oz} \end{pmatrix} \cdot \begin{pmatrix} -\alpha_x \cdot {}_kV_x + \beta_x\left(\psi_{xx}^k - \psi_{xy}^k\right) \\ -\alpha_y \cdot {}_kV_y + \beta_y\left(\psi_{yy}^k + \psi_{yx}^k\right) \\ -\alpha_z \cdot {}_kV_z + \beta_z\left(\psi_{zz}^k\right) \end{pmatrix} \tag{14}$$

and the following normalized admittances:

$$Y_{ox} = \frac{4}{\beta_x - 4}, \tag{15}$$

$$Y_{oy} = \frac{4}{\beta_y - 4}, \tag{16}$$

$$Y_{oz} = \frac{4}{\beta_z - 4}. \tag{17}$$

The discrete convolutions appearing in the set of (14) can be evaluated recursively due to the exponential nature of the complex time domain susceptibility functions.

In order to implement the TLM algorithm, we compute the voltages sources using (14). These voltages are injected into ports (16, 17, 18) of the HSCN. Then the reflected pulses in the node's lines are computed from the equivalent voltages and currents. At this stage, the connection procedure among the nodes is launched.

III. Numerical Results

In order to validate the HSCN-TLM approach, we compute the reflection and the transmission coefficients for a magnetized plasma layer, which has been excited using a normally incident Gaussian plane wave. The problem space considered had 350 Δl in z direction, where Δl is the mesh width taken to be $\Delta l = 75\ \mu$m. The magnetized plasma layer occupying cells 200–320 corresponding to a length of 9 mm. This plasma is characterized by: the plasma frequency $\omega_p = 2\pi 50.00E9$ rad/s, the cyclotron frequency $\omega_b = 2\pi 47.75E9$ rad/s and the electron collision frequency $\nu_c = 2\pi 3.18E9$ rad/s. Since the incident plane wave is linearly polarized along *y* direction, the reflection coefficients for left-hand circularly polarized (LCP) and right-hand circularly polarized (RCP) waves versus the frequency were obtained by using

$$R_{\mathrm{LCP}} = E_y^r(\omega) - jE_x^r(\omega) \tag{18}$$

$$R_{\mathrm{RCP}} = E_y^r(\omega) + jE_x^r(\omega). \tag{19}$$

The transmission coefficients were computed in the same way.

Figs. 1 and 2 illustrate the good agreement between the TLM results and those computed analytically considering RCP and LCP polarizations for these reflection and transmission coefficients respectively.

Fig. 3 depicts the transmitted field for plasma layers considering the following lengths: 100 Δl, 200 Δl, 300 Δl, and 400 Δl and a problem space of 700 Δl in z-direction. The obtained results illustrate clearly the Faraday rotation effect of a single-frequency linearly polarized plane wave [13].

$${}_{k+1}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} 1 & C_{xy} & 0 \\ -C_{yx} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} {}_kE_x - \alpha_x \cdot {}_kE_x + \beta_x\left(\psi_{xx}^k - \psi_{xy}^k + \frac{\Delta t}{\varepsilon_0}{}_{k+1/2}(\nabla \times H)_x\right) \\ {}_kE_y - \alpha_y \cdot {}_kE_y + \beta_y\left(\psi_{yy}^k + \psi_{yx}^k + \frac{\Delta t}{\varepsilon_0}{}_{k+1/2}(\nabla \times H)_y\right) \\ {}_kE_z - \alpha_z \cdot {}_kE_z + \beta_z\left(\psi_{zz}^k + \frac{\Delta t}{\varepsilon_0}{}_{k+1/2}(\nabla \times H)_z\right) \end{pmatrix} \tag{2}$$

$${}_{k+1}\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} 1 & C_{xy} & 0 \\ -C_{yx} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} {}_kV_x + \frac{1}{4+Y_{ox}}\left({}_{k+1}V_{svx} + {}_kV_{svx}\right) + \frac{4}{4+Y_{ox}}\frac{\Delta t}{\varepsilon_0}{}_{k+1/2}(\nabla \times H)_x \\ {}_kV_y + \frac{1}{4+Y_{oy}}\left({}_{k+1}V_{svy} + {}_kV_{svy}\right) + \frac{4}{4+Y_{oy}}\frac{\Delta t}{\varepsilon_0}{}_{k+1/2}(\nabla \times H)_y \\ {}_kV_z + \frac{1}{4+Y_{oz}}\left({}_{k+1}V_{svz} + {}_kV_{svz}\right) + \frac{4}{4+Y_{oz}}\frac{\Delta t}{\varepsilon_0}{}_{k+1/2}(\nabla \times H)_z \end{pmatrix} \tag{13}$$

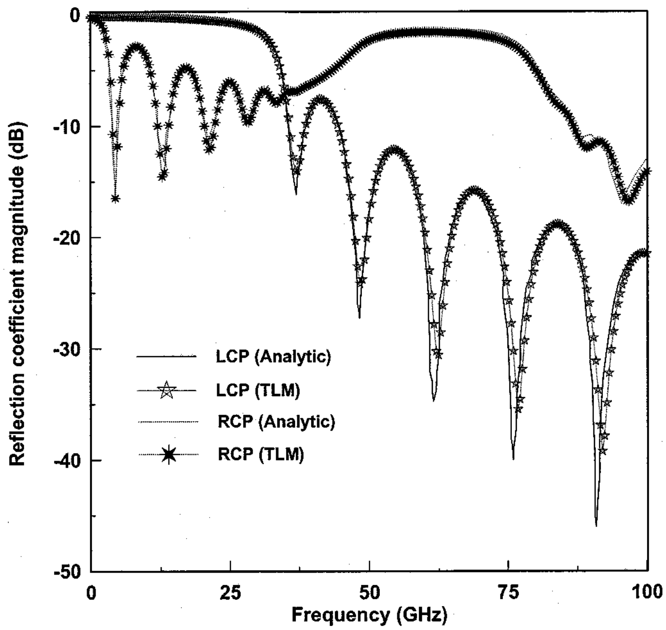

Fig. 1. Reflection coefficients of magnetized plasma layer versus frequency.

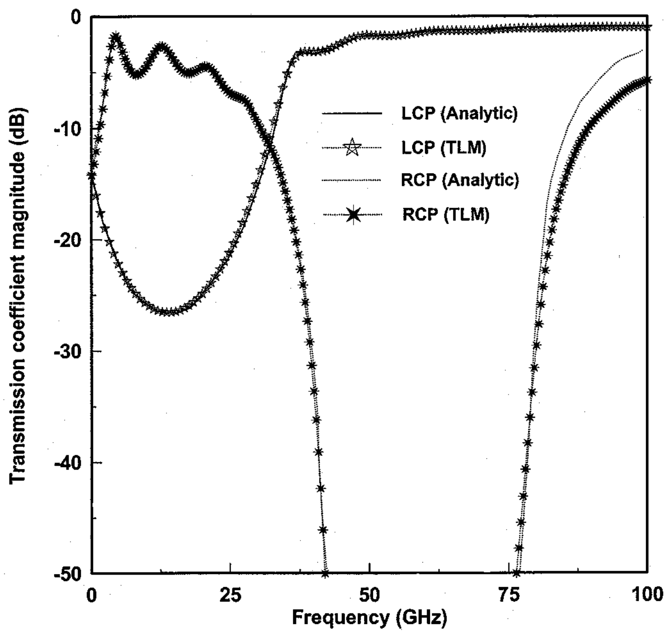

Fig. 2. Transmission coefficients of magnetized plasma layer versus frequency.

IV. Conclusion

In this paper, a substantial development of the TLM method, based on the hybrid symmetrical condensed node (HSCN) and voltage sources, has been proposed to model frequency dependent anisotropic plasma. The technique was validated by the study of EM wave reflection and transmission through a magnetized plasma layer. Excellent agreement between our results and the theories ones reflects the efficiency of the proposed model.

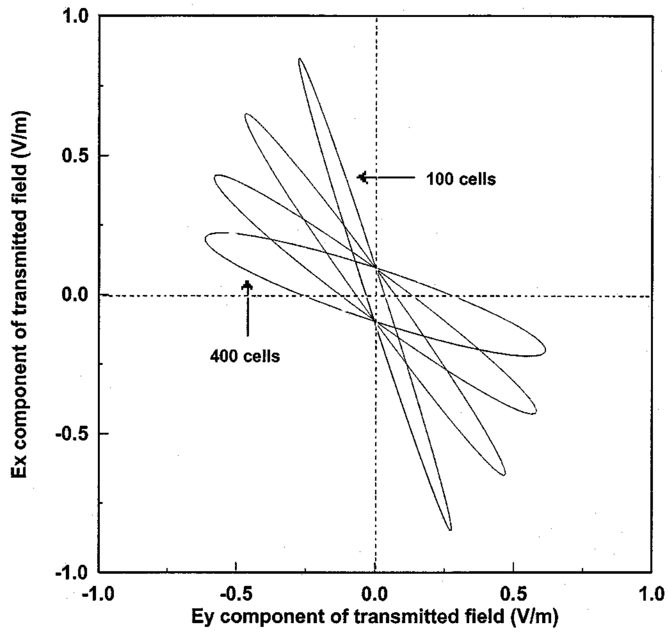

Fig. 3. Faraday rotation effect in a magnetized plasma layer at a frequency of 91 GHz.

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